UPPSALA UNIVERSITET
Matematiska Institutionen
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VT22
Assignment 2
2022/05/09

Partial Differential Equations with Applications to Finance

Instructions: There are two problems each worths 10 points. A score of 12 points yield 1 bonus point in the final grade. Your answer should be well motivated in order to receive full credit in each question.

Please submit your solutions under "Assignment 2" before midnight 31st May. The submission can either be handwritten or typed, but it should be in .pdf format. You should expect 5 working days for your submission to be corrected.

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1. Solve the problem of minimizing

$$\mathbb{E}_x[\exp\{\int_0^T u_t^2 dt + X_T^2\}]$$

given the dynamics

$$dX_t = u_t\, dt + \sigma dB_t,$$

where the control u_t is Markovian and there are no control constraints.

Hint: *Make the ansatz*

$$V(t,x) = e^{A(t)x^2 + B(t)}$$

and use the result from Exercise 904.

2. Solve the optimal stopping problem

$$V(x) = \sup_\tau \mathbb{E}[e^{-\beta\tau} B_\tau^2],$$

where B is a standard Brownian motion.

Hint: *The continuation region is presumably of the form $(-b, b)$. Find an (unsolvable) equation for b.*