Partial Differential Equations with Applications to Finance

VT 2022

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1 General information

- **Lectures:** All lectures and exercise sessions will be given in person at Ångström. There will be 12 lectures. Regular attendance is essential but not required.
- **Exercise sessions:** There will be 3 exercise sessions, during which you can present solutions to certain exercises in order to earn bonus points. We will solve the rest of the exercises together.
- **Office Hours:** You are welcome to visit me at Å14209 upon request. There will be one scheduled drop-in Q&A session before the written exam.
- **Announcements:** I will be using the Studium *"Announcement"* feature as the main communication tool. Please make sure to go under *"Account - Notifications"* to set up the announcement notifier so that you do not miss any important information. I will also send out emails via Studium *"Inbox"* to remind you about the important dates.
- **Administrative issues:** For course registration, exam registration, retakes and other specific administrative issues please contact the administration office (studexp@math.uu.se) or your study counselor.

2 Course content

- **Objectives:** This course is primarily designed for graduate students in mathematics. We will discuss the basics of stochastic processes, stochastic calculus, SDEs and their connection to parabolic PDEs, and further discuss topics in optimal stopping and stochastic control theory. Like most of your courses, we will make statements and prove theorems. However, our goal in this course is not to learn the definitions and details of everything. Instead of formulating the theories step by step, we will look at the general mathematical mechanisms of those theories, how different aspects are surprisingly correlated, formulate concrete examples and solve them. By the end of this course we will hopefully be able to know what kind of properties there are with SDEs, what questions to ask, and how one solves them.
  This course is, generally speaking, a probability course. We will not learn how to solve different kinds of PDEs, and we will not do numerics.
- **Prerequisites:** Working knowledge of (measure theory based) probability theory and a general understanding of (parabolic) PDEs. Knowledge on stochastic calculus and financial derivatives would be a plus but not necessary.
- **Tentative Course Plan:** The course consists of five modules as shown below, and each module will be associated with one or two basic examples in financial mathematics. Please check the last page of this document for the detailed preliminary lecture plan.

Module 1. A review of stochastic processes and Ito calculus
Module 2. Feynman-Kac theorem and its applications
Module 3. The heat equation and the Kolmogorov equations
Module 4. Optimal control theory
Module 5. Optimal stopping and free boundary problems

3 Course Literature

There is no course book. The course material is mainly based on the following books and lecture notes.

- Robert Kohn "PDE for Finance", Spring 2015. Online course notes here.
- Øksendal, B. (2014). Stochastic Differential Equations: An Introduction with Applications (Universitext). Springer. ISBN: 3540047581
- Björk, T. (2005). Arbitrage theory in continuous time. Oxford [u.a.]: Oxford Univ. Press. ISBN: 0199271267

4 Course Assessment

- **Assignments:** There will be two non-mandatory assignments, each worths 1 bonus point. The goal of the assignments is to introduce you to exam-like questions. I will grade them as if they were exam questions, so that as feedback you can see how I will be grading exam questions. Collaboration is encouraged. However, each student should submit their solutions independently. *You can get at most* 2 *bonus points from the hand-in assignments.*
- **Presentation of solutions:** I will upload a list of recommended exercises including some exercises we will solve during the three exercise sessions. You are strongly encouraged to register and present your solutions on the board during these sessions. The exercises are taken mainly from previous year exams, assignments and examples. This is a good exercise for us to not only understand the solutions, but be able to explain them to others pedagogically.
  To register the problem you want to present, go to this page on Studium and write down your name next to the problems. We will have the presentation in chronological order. You are very welcome to ask for hints and discuss with me before presenting your solutions.
  Each problem you correctly present gives 1 bonus point. *You can get at most* 2 *bonus points for the presentations.*
- **Written exam:** There will be one mandatory closed-book exam on *June 2nd, 08.00-17.00.* You will get the exam paper with 5 problems each worths 8 points.
  You can get at most 40 *points from the written exam.*
- **Retakes:** The retakes are scheduled in August and Easter.
  OBS. Bonus points cannot be used for the retakes.

The final grade is calculated as follows:

- $18 - 24$ points: grade 3
- $25 - 31$ points: grade 4
- $32 - 44$ points: grade 5

5 Course Evaluation

There will be three anonymous surveys, the **pre-course survey**, the **mid-course evaluation** and **course evaluation** on Studium. All course evaluations are non-mandatory, but they help all of us identify what is working and what could use improvement. It is therefore recommended that you take some time and answer the surveys.

6 Preliminary Lecture Plan:

Module 1.
F1. Course introduction. Important concepts regarding stochastic processes 22/03
F2. The Ito integrals and their properties 24/03
F3. Stochastic differential equations and the Ito's formula 28/03
Module 2.
F4. The infinitesimal generator and the Feynman-Kac Theorem 30/03
F5. Brownian motion and applications of the FK theorem 06/04
L1. Exercise session I 08/04
Assignment I 19/04
Module 3.
F6. The heat equation and its properties 20/04
F7. The heat equation II, Kolmogorov equations for BM 22/04
F8. Markov processes and the Kolmogorov equations 26/04
L2. Exercise session II 03/05
Module 4.
F9. Optimal control problems, the Hamilton-Jacobi-Bellman Equation 06/05
F10. Merton's portfolio allocation problems 10/05
Module 5.
F11. Optimal stopping theory I 12/05
F12. Optimal stopping theory II, the American options 17/05
L3. Exercise session III 20/05
Q&A session ..... Week 21
Assignment II 31/05
Exam 02/06